dyadic Green's function to be calculated, and circumvents the extensive algebraic manipulation associated with formulations described previously. Numerical results obtained using this method have been presented and compared to other existing data. Good agreement was obtained in all cases thus establishing the accuracy and applicability of the method for the full range of structure parameters. Design curves have been included here for millimeter-wave fin lines of practical interest. Both center and off-center fin locations have been discussed, and the off-center location was shown to result in no significant change in impedance for small values of W/b. Lower impedance may be realized, however, by using a single-fin configuration.

It is clear from the results presented here that the fin line may exhibit the characteristics of a ridged waveguide, slotline, or dielectric slab-loaded waveguide, depending upon the values of the various fin-line parameters. All of these structures are fin-line substructures.

References

[1] P. J. Meier, "Integrated fin-line millimeter components," *IEEE Trans. Microwave Theory Tech.* vol. MTT-22, pp. 1209–1216, Dec. 1974.

[2] ——, "Millimeter integrated circuits suspended in the *E*-plane of rectangular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 726–733, Oct. 1978.

[3] ——, "Printed circuit balanced mixer for the 4 and 5 mm bands," in *IEEE MTT-S Symp. Dig.*, pp. 84–86, May 1979.

[4] W. Kpodzo *et al.*, "A quadriphase modulator in fin-line technique," in *IEEE MTT-S Symp. Dig.*, pp. 119–121, May 1979.

[5] A. M. K. Saad and K. Schunemann, "A simple method for analyzing fin-line structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 1002–1007, Dec. 1978.

[6] W. Hoefer, "Fin-line parameters calculated with the TLM method," in *IEEE MTT-S Symp. Dig.*, pp. 341–343, May 1979.

[7] T. Itoh, "Spectral domain analysis of dominant and higher order modes in fin-lines," in *IEEE MTT-S Symp. Dig.*, pp. 344–345, May 1979.

[8] T. Itoh and R. Mittra, "Dispersion characteristics of slot lines," *Electron. Lett.*, vol. 7, pp. 364–365, July 1971.

[9] ——, "Spectral-domain approach for calculating the dispersion characteristics of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, (Short Papers), vol. MTT-21, pp. 496–499, July 1973.

[10] ——, "A technique for computing dispersion characteristics of shielded microstrip lines," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-22, pp. 896–898, Oct. 1974.

[11] J. B. Knorr and K -D. Kuchler, "Analysis of coupled slots and coplanar strips on dielectric substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 541–548, July 1975.

[12] J. B. Knorr and A. Tufekcioglu, "Spectral-domain calculation of microstrip characteristic impedance," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 725–728, Sept. 1975.

[13] S. Hopfer, "The design of ridged waveguides," *IRE Trans. Microwave Theory Tech.*, vol. MTT-3, pp. 20–29, Oct. 1955.

[14] R. O. Lagerlöf, "Ridged waveguide for planar microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 499–501, July 1973.

[15] R. Collin, *Field Theory of Guided Waves.* New York: McGraw-Hill, 1960.

[16] P. H. Vartanian *et al.*, "Propagation in dielectric slab loaded rectangular waveguide," *IRE Trans. Microwave Theory Tech.*, vol. MTT-6, pp. 215–222, Apr. 1958.

[17] E. A. Mariani *et al.*, "Slot line characteristics," *IEEE Trans. Microwave Theory Tech.* (1969 Symp. Issue), vol. MTT-17, pp. 1091–1096, Dec. 1969.

The Accuracy of TLM Analysis of Finned Rectangular Waveguides

YI-CHI SHIH, STUDENT MEMBER, IEEE, AND WOLFGANG J. R. HOEFER, SENIOR MEMBER, IEEE

***Abstract*—This paper investigates three sources of error affecting the Transmission Line Matrix (TLM) analysis of finned rectangular waveguides. It is shown how truncation and velocity errors can be minimized, and a diagram for maximum coarseness error affecting the TLM analysis is presented. After error correction, cutoff frequencies obtained with the TLM method are in excellent agreement with results obtained with the Transverse Resonance Method.**

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The authors are with the Department of Electrical Engineering, University of Ottawa, Ottawa, Ont., Canada K1N 6N5.

I. Introduction

The TWO-DIMENSIONAL Transmission Line Matrix (TLM) method was developed by Johns and Beurle [1] and has been successfully applied to waveguide bifurcation scattering problems [1] and to the ridged waveguide problem [2].

It is a powerful tool for solving the homogeneous wave equation in complex structures and, therefore, can be used to verify the accuracy of approximate solutions, provided that the errors affecting the TLM solution are known and can be corrected. The aforementioned authors have

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TABLE I
COMPARISON OF VALUES OBTAINED WITH THE TLM METHOD FOR THE NORMALIZED CUTOFF FREQUENCY OF THE TE_{10} MODE IN RECTANGULAR WAVEGUIDES USING DIFFERENT MESH SIZES[a]

Number of nodes along side b	Assumption: $v = c/\sqrt{2}$ as proposed in [1]		Assumption: v as defined by eq. (3)		Accurate value of
$b/\Delta\ell$	Normalized TE_{10} cutoff frequency b/λ_c	Velocity plus truncation errors $E_V + E_T$ (%)	Normalized TE_{10} cutoff frequency b/λ_c	Velocity plus truncation errors $E_V + E_T$ (%)	b/λ_c
1	0.23570	-5.7	0.25003	0.01	
2	0.24667	-1.3	0.24996	-0.02	
4	0.24924	-0.3	0.25005	0.02	0.25
8	0.24974	-0.1	0.24994	-0.02	
16	0.24981	-0.08	0.24986	-0.06	

[a]The truncation error E_T is always smaller than 0.2 percent. The velocity error E_V can be avoided by considering that the propagation velocity along the axes of the TLM mesh is governed by (3).

specified three types of errors affecting the results obtained with this method, namely, truncation error, velocity error, and "coarseness" error. While the first two types of errors can be readily defined and predicted, the coarseness error which reflects the spacial resolution of the TLM is more difficult to evaluate and to analyze.

The present paper investigates these three errors as they occur in the TLM analysis of a finned waveguide (a ridged waveguide with a centered ridge of zero thickness). The finned waveguide was selected for the following reasons:

a) This structure was used successfully by Konishi *et al.* [4], [5] in the realization of converters and filters. It is thus of practical interest.

b) It can be analyzed very accurately using the Transverse Resonance Method and Marcuvitz' [6] expression for the susceptance of the fins. Results can thus be verified with accuracy.

c) This structure is a special case of the fin line, a very promising transmission medium for millimeter waves. The authors plan to determine the parameters of discontinuities in both finned waveguides and fin lines with the TLM method. These structures are not easily accessible through other methods.

The present paper thus establishes the suitability of the TLM method as a verification of other methods and as a tool for millimeter wave circuit design.

The reader may ask why the cutoff wavelength is evaluated rather than the guided wavelength at some higher frequency. The answer is that the guided wavelength λ_g can be obtained from the cutoff wavelength λ_c by the well-known expression

$$\lambda_g = \lambda\left[1-(\lambda/\lambda_c)^2\right]^{-1/2} \tag{1}$$

λ = free space wavelength.

II. TRUNCATION ERROR AND VELOCITY ERROR

When the resonant frequency of a structure is evaluated with the TLM method, the impulse response of a transmission line network simulating this structure is computed. In a practical calculation this impulse response must be limited in time. The error introduced by this limitation is the truncation error E_T. Its maximum value is determined [2] by

$$E_T = \frac{\Delta S}{\Delta l/\lambda_c} = \pm\frac{3\lambda_c}{SN^2\pi^2\Delta l} \tag{2}$$

where S is the frequency separation (expressed in terms of $\Delta l/\lambda$) between two neighboring peaks in the frequency response obtained from the impulse response via Fourier transform. N is the number of iterations processed. Δl is the mesh parameter of the transmission line lattice, and λ is the free space wavelength. Normally, the truncation error is smaller than the maximum value obtained with the above formula. It decreases rapidly with an increasing number of iterations.

Due to the slow wave characteristics of the transmission line lattice, the velocity of a wave traveling through this network depends on the angle between the propagation direction and the mesh axes. By assuming, as Johns and Beurle [1] suggested, that the propagation velocity is independent of the direction of propagation and equal to $c/\sqrt{2}$, the velocity error E_V is introduced.

If the wave propagates along one of the axes of the mesh, its propagation velocity is defined by

$$\sin\left(\frac{\beta_n\Delta l}{2}\right) = \sqrt{2}\,\sin\left(\frac{\omega\Delta l}{2c}\right). \tag{3}$$

$\beta_n = 2\pi/\lambda_n$, where λ_n is the wavelength of propagation along this axis. Thus if a rectangular waveguide is treated with the TLM method, the velocity error can be eliminated in the case of the TE_{10} mode by determining the wave velocity in the direction of the broad wall directly from (3).

To demonstrate the effectiveness of the correction of the velocity error, Table I lists values for the cutoff frequency of the TE_{10} mode in a rectangular waveguide of aspect ratio $b/a = 1/2$, obtained with the TLM method using different mesh sizes. In all calculations, the number of iterations was sufficiently high to keep the truncation error below 0.2 percent. It is found that by assuming a velocity of $c/\sqrt{2}$, the coarsest mesh yields a cutoff frequency which is too low by 5.7 percent. If, however, (3) is applied to correct the velocity error, the remaining inaccuracy is negligible and may be attributed to truncation (<0.2 percent).

Even in finned rectangular waveguides, the velocity error in the TE_{10} cutoff frequency may be practically eliminated by applying (3), in spite of the fact that in the immediate vicinity of the fins, the wave propagation does not exactly coincide with a mesh axis. However, a third type of error (coarseness error) is dominant in the TLM calculations of finned waveguides. This error will be defined and evaluated below.

III. Coarseness Error

It has been shown in the previous section that the TLM evaluation of the fundamental cutoff frequency in rectangular waveguides is quite accurate, even if a very coarse mesh is used, provided that the velocity error is corrected. However, if transverse discontinuities (e.g. ridges or fins) are present in the guide, the TLM mesh must be fine enough to resolve the fields in the regions where the gradient of the electric potential is highest. The question is, in other words, how many nodes should be chosen in the immediate vicinity of the discontinuity in order to keep the coarseness error within desired limits?

Unfortunately, there are no general rules or equations to evaluate this error. In order to study its behavior in the case of finned rectangular waveguides, the normalized TE_{10} cutoff frequency b/λ_c of the guide shown in Fig. 1 was calculated using several mesh sizes. An aspect ratio of $b/a=1/2$ was chosen, and several values of $b/\Delta l$ for the mesh parameter were selected. The corresponding TE_{10} cutoff frequencies are presented in Fig. 2.

Note that in all cases, the error due to truncation is smaller than 0.4 percent, and the velocity error is negligible after corrections according to (3). Circles represent the results obtained for a normalized gap width of $d/b=0.5$, and triangles correspond to $d/b=0.25$.

It appears that the normalized cutoff frequency of the fundamental mode tends asymptotically toward its accurate value as the mesh becomes finer. It turns out that the coarseness error, i.e., the relative separation between the calculated and the asymptotic value is directly proportional to $\Delta l/b$, the inverse of the abscissa in Fig. 2. It is therefore easy to obtain accurate values for b/λ_c by linear extrapolation toward $\Delta l/b=0$.

Table II shows how the asymptotic values of b/λ_c depend on the normalized gap width d/b. For comparison, cutoff frequencies obtained with the Transverse Resonance Method [3] are also shown. Excellent agreement exists between both methods once the coarseness error has been eliminated by extrapolation.

Furthermore, the dependence of the coarseness error on the number of nodes situated within the gap separating the fins has been investigated. (The error was always evaluated with respect to the asymptotic value of the cutoff frequency.) Fig. 3 shows this error in percent as a function of the normalized gap width d/b. The waveguide

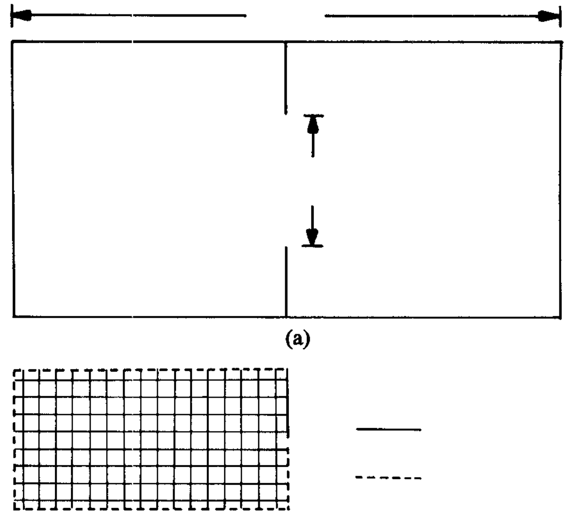

Fig. 1 (a) Cross section of finned rectangular waveguide. (b) Two-dimensional shunt node TLM network simulating the waveguide shown in (a). Through introduction of appropriate symmetry conditions, only one quarter of the cross section is required for the analysis of the TE_{10} mode. Note that in the TLM network, boundaries are dual to those in the real structure. (——: Electric wall;----: magnetic wall.)

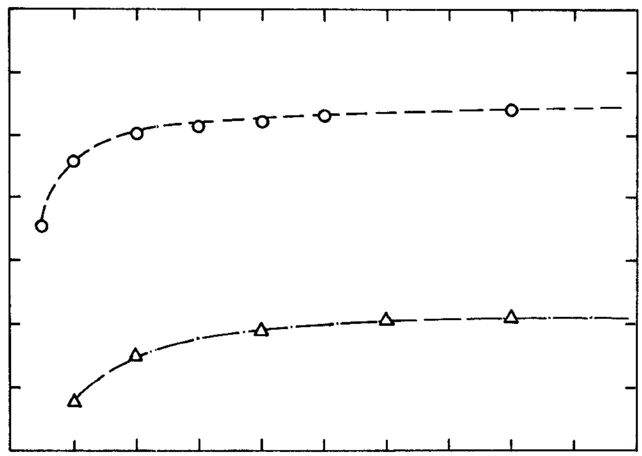

Fig. 2 Normalized TE_{10} cutoff frequency in rectangular finned waveguides of aspect ratio $b/a=1/2$ with normalized gap widths of $d/b=1/2$ (circles) and 1/4 (triangles), obtained with the TLM method using increasingly fine meshes.

TABLE II
Normalized TE_{10} Cutoff Frequency in Finned Waveguide of Aspect Ratio $b/a=1/2$, for Several Normalized Gap Widths d/b[a]

Normalized Gap Width d/b	Normalized TE_{10} cutoff frequency b/λ_c		Difference in %
	TLM method	Transverse Resonance Method	
2/3	0.2391	0.2389	0.08
1/2	0.2253	0.2249	0.19
1/3	0.2054	0.2052	0.10
1/4	0.1932	0.1928	0.21
1/8	0.1697	0.1690	0.41
1/16	0.1522	0.1518	0.28

[a]TLM results have been obtained by extrapolation toward $\Delta l/b=0$, and they are compared with values obtained with the Transverse Resonance Method [3].

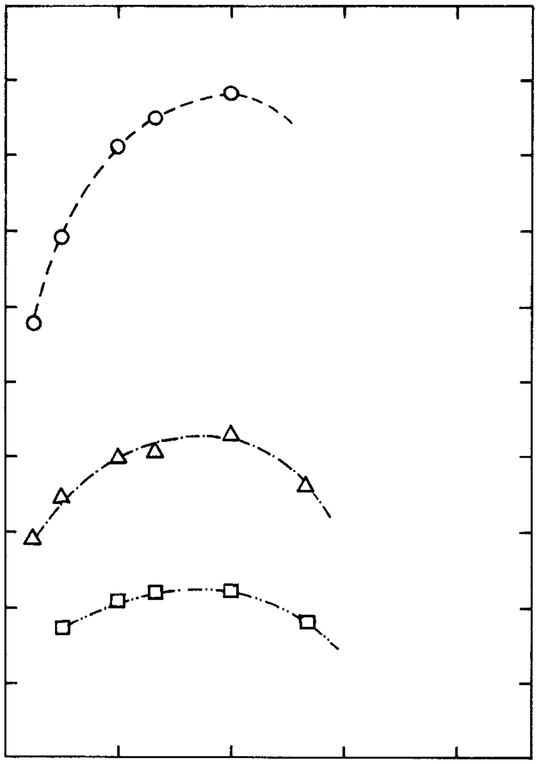

Fig. 3 Coarseness error affecting the normalized TE_{10} cutoff frequency in finned waveguides of aspect ratio $b/a=0.5$ as a function of the gap width, when calculated with the TLM method. Two, four, and eight nodes have been placed within the gap.

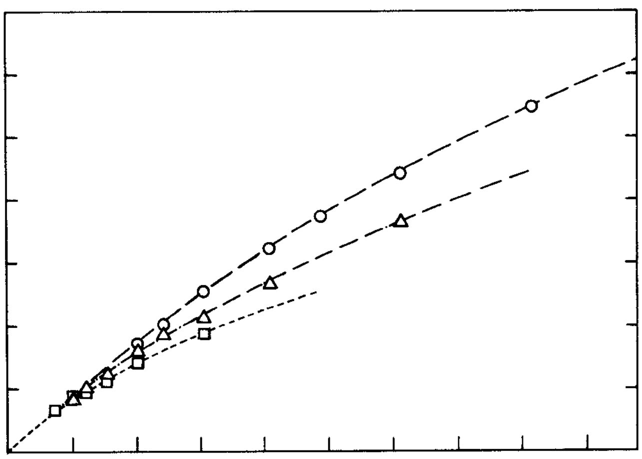

Fig. 4 Maximum coarseness error affecting the normalized TE_{10} cutoff frequency in finned waveguides as a function of $\Delta l/a$ when calculated with the TLM method. Two, four, and eight nodes have been placed within the gap.

aspect ratio was kept constant at $b/a=1/2$, and calculations have been made with two, four, and eight nodes situated between the fins. It appears that whatever the coarseness of the TLM mesh, the error is always maximum in the vicinity of $d/b=1/2$, which is henceforth considered to be the "worst case."

Assuming this worst case to prevail, the coarseness error was then evaluated for various values of $\Delta l/a$ and presented in Fig. 4. Since d/b is constant (equal to 1/2), the circles represent the maximum coarseness error for finned guides of various aspect ratios b/a, all featuring two nodes between the fins. Triangles correspond to four nodes between fins, and squares to eight nodes. Hence, it is possible to deduce from Fig. 4 the maximum coarseness error when calculating the cutoff frequency of the lowest mode of propagation in a finned rectangular waveguide with the TLM method.

IV. Features of the Computer Program

The TLM program is written in Fortran IV and requires 4 locations to represent one node. The storage array required for a 16×8 matrix (see Fig. 1) is thus $4\times16\times8$ locations. An additional number of locations, equal to the number N of iterations processed (usually $N\approx800$), is required to store the output impulse function. The reserved arrays and the program itself require a total core memory of about 10K bytes. An IBM 360 System executes 800 iterations in about 8 s of CPU-time.

V. Conclusion

When the cutoff frequency of the TE_{10} mode in finned waveguides is calculated with the TLM method, truncation, velocity, and coarseness errors occur.

The truncation error can be evaluated easily using (2). The velocity error can be corrected by applying (3). The coarseness error, which is dominant in the structures discussed here, was found to be maximum for a normalized gap width of $d/b=1/2$, regardless of the aspect ratio b/a and the coarseness of the TLM mesh. Its value can be predicted with the aid of Fig. 4 for a wide range of guide dimensions and mesh sizes. More important, the coarseness error can be eliminated by linearly extrapolating results obtained with lattices of different mesh parameters. For reasons of geometrical similarity, this study is relevant to the TLM analysis of fin lines. They differ from finned waveguides only by the presence of a thin dielectric sheet of low permittivity adjacent to the fins.

References

[1] P. B. Johns and R. L. Beurle, "Numerical solution of two-dimensional scattering problems using a transmission-line matrix," *Proc. Inst. Elec. Eng.*, vol. 118, no. 9, Sept. 1971.

[2] P. B. Johns, "Application of transmission-line-matrix method to homogeneous waveguides of arbitrary cross-section," *Proc. Inst. Elec. Eng.*, vol. 119, no. 8, Aug. 1972.

[3] W. J. R. Hoefer, "Fin line design made easy," presented at the *1978 IEEE-MTT-S Int. Microwave Symp.*, June 27–29, Ottawa, Ont., Canada.

[4] Y. Konishi and K. Uenakada, "The design of a bandpass filter with inductive strip-planar circuit mounted in waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 869–873, Oct. 1974.

[5] Y. Konishi, "Planar circuit mounted in waveguide used as a downconverter," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 716–719, Oct. 1978.

[6] N. Marcuvitz, *Waveguide Handbook*, M.I.T. Radiation Laboratory Series no. 10. Boston, MA: Boston Technical Publishers, Inc., 1964.